

Abstracts of short communications

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Inertial automorphisms of an abelian group

U. Dardano

We analyze into details inertial automorphisms of abelian groups. An automorphism γ of a group G is said *inertial* iff $H \cap H^\gamma$ has finite index in both H and H^γ , for each subgroup H of G . Casolo considered automorphisms γ with the property that for each $H \leq G$ there is K such that $H \leq K^\gamma = K \leq G$ and $|K : H|$ is finite. Later, Franciosi – de Giovanni – Newell considered the dual property with $K = K^\gamma \leq H$ and $|H : K|$ finite (they call such a \mathfrak{g} an almost power automorphism). Inertial automorphisms seem to be the framework to regard those two properties from a unique point of view. Recently there has been much interest in inertial groups, that is groups in which all inner automorphisms are inertial. As an example, we recall that D. Robinson characterized such groups in generalized finiteness and solvability conditions. Also recall that FC-groups are inertial.

The work is by U.Dardano and S.Rinauro.

Characterization of finitely generated infinitely iterated wreath products

E. Detomi

Given a sequence of $(G_i)_{i \in \mathbb{N}}$ of finite transitive groups of degree n_i , let W be the inverse limit of the iterated permutational wreath products $G_m \wr \cdots \wr G_2 \wr G_1$. We prove that W is (topologically) finitely generated if and only if $\prod_{i=1}^{\infty} (G_i/G'_i)$ is finitely generated and the growth of the minimal number of generators of G_i is bounded by $d \cdot n_1 \cdots n_{i-1}$ for a constant d . Moreover we give a criterion to decide whether W is positively finitely generated.

The work is by E. Detomi and A. Lucchini.

Some algorithms to identify in GAP properties related to permutability in finite soluble groups

R. Esteban-Romero

In this talk we present some algorithms which will allow us to decide with the help of the computer algebra system GAP whether a group is Dedekind (that is, it has all subgroups normal) or Iwasawa (that is, it has all subgroups permutable). We also present algorithms to check whether a soluble group is a T-group (normality is transitive), a PT-group (permutability is transitive), or a PST-group (Sylow permutability is transitive).

Groups with restrictions on automorphism group

D. Imperatore

Let G be a group. It is a natural (and usually hard) problem to solve the equation $\text{Aut } X \simeq G$, i.e., to determine all groups X having G as their automorphism groups (up to isomorphisms). In this talk we present some results on this subject.

Detecting properties of groups from subgroups with few generators

M. Morigi

Let G be a finite group and let $i(G)$ be a property of G . When $i(G)$ is the exponent, or the prime graph, or the set of (isomorphism classes of) composition factors of G , we show that there is a subgroup H of G with few generators such that $i(H) = i(G)$. Moreover, if C is a subgroup of G , we can detect the primes dividing the index of C in G by looking at $|H : C \cap H|$, where H is a subgroup of G with at most 3 generators.

Classifying semisimple orbits of θ -groups

F. Oriente

θ -groups form a wide class of reductive algebraic groups, for which there is a well-developed theory of their orbits. This makes it possible to attempt to classify them. In the proposed presentation we will address the case of closed orbits. In the past decades this has been done in separate cases by several authors (Elashvili-Vinberg, Galitski-Timashev, Pervushin). We develop a more systematic approach using computational techniques. For this we describe an algorithm for finding a Cartan subspace, and the corresponding little Weyl group. The invariants of this little Weyl group, acting on the Cartan subspace, classify the closed orbits. Using these methods we have obtained the little Weyl groups for most θ -groups arising from the simple Lie algebras of exceptional type.

This is joint work with Willem de Graaf.

On the Weil representations of symplectic and unitary groups

C. A. Pallikaros

In this joint work with A. Zalesski we study the restriction of the Weil representations of symplectic and unitary groups to subgroups that are the centralizers of certain elements, and show that these are multiplicity free. This work is along the line of the Howe philosophy for so called dual pairs in the symplectic and unitary groups.

On some properties of generalized FC -groups

E. Romano

It is well known that groups with finite conjugacy classes (i.e. FC -groups) can be considered as the most natural tool in order to study properties which are common both to finite groups and abelian groups. The attempt to investigate wider classes of groups generalizing both finiteness and nilpotency suggests the introduction of a new group theoretical property. In [2] the authors started the study of a class of generalized FC -groups defined recursively as follows: FC^0 is the class of finite groups and a group G belongs to the class FC^{n+1} if $G/C_G(\langle x \rangle^G) \in FC^n$ for all x in G . Thus FC^1 -groups are precisely groups with finite conjugacy classes and the class FC^n obviously contains all finite groups and all nilpotent groups with class at most n . Clearly, every FC^n -group is FC -nilpotent. Many investigations on FC^n -groups naturally generalize those for FC . For instance, in [3] results on Sylow Theory and also the full description of ERF -groups are extended from FC -groups to FC^n -groups (recall that G is said to be extended residually finite, briefly ERF , if every subgroup is closed in the profinite topology, i.e., if every subgroup of G is an intersection of subgroups of finite index). In this talk, we want to extend to FC ([4]) the theory of pronormality developed for FC -groups in [1].

- [1] F. de Giovanni, G. Vincenzi: *Pronormality in infinite groups*, Proc. Roy. Irish Acad., 100A (2000), 189–203.
- [2] F. De Giovanni, A. Russo, G. Vincenzi: *Groups with restricted conjugacy classes*, Serdica Math. J. 28, 3 (2002), 241–254.
- [3] D. J. S. Robinson, A. Russo, G. Vincenzi: *On the theory of generalized FC -groups*, J. Algebra, (2009), doi:10.1016/j.jalgebra.2009.04.002.
- [4] E. Romano, G. Vincenzi: *Pronormality in generalized FC -groups*, to appear on Bull. of the Austral. Math. Soc.

Chains of weakly closed p -subgroups

L. Serena

Let G be a finite group and P be a Sylow p -subgroup of G . Suppose that V is a strongly closed subgroup of P . Several years ago we gave a necessary and sufficient condition for V to control the strong fusion. This result deals with central chains of weakly closed subgroups of V .

Using a combination of this result and a Lemma of Laffey, we generalize and unify several recent results proved by various authors on the existence of a normal p -complement and supersolubility of G . Further results on this topic are obtained by using the concept of Φ -chain, which is a particular chain related to the Frattini subgroup of V .

This work is by A. L. Gilotti and L. Serena.

Groups with soluble non-normal subgroups

A. Tortora

In this talk, based on joint work with K. Ersoy and M. Tota, we deal with the structure of locally graded groups whose non-normal subgroups are soluble.

p -Groups with few Conjugacy Classes of Normalizers

M. Tota

Let $\nu(G)$ be the number of conjugacy classes of non-normal subgroups of a group G . In [1], R. La Haye and A. Rhemtulla proved that if G is a finite p -group with $\nu(G)$ strictly greater than 1, then $\nu(G)$ is at least p . Now, denote by $\omega(G)$ the number of conjugacy classes of normalizers of a finite p -group G . In a joint work with R. Brandl and C. Sica, we look for a similar bound for $\omega(G)$ in terms of p .

- [1] R. La Haye, A. Rhemtulla: *Groups with a Bounded Number of Conjugacy Classes of Non-normal Subgroups*, J. Algebra 214, (1999), 41–63.

On minimal non- M_rC -groups

N. Trabelsi

A group G is said to be a minimax group if it has a finite series whose factors satisfy either the minimal or the maximal condition. Let $D(G)$ denotes the subgroup of G generated by all the Chernikov divisible normal subgroups. If G is a soluble-by-finite minimax group and if $D(G) = 1$, then G is said to be a reduced minimax group. Also G is said to be an M_rC -group if $G/C_G(x^G)$ is a reduced minimax group for all $x \in G$. This is a generalisation of the familiar property of being an FC -group. Finally, if \mathfrak{X} is a class of groups, then G is said to be a minimal non- \mathfrak{X} -group if it is not an \mathfrak{X} -group but all of whose proper subgroups are \mathfrak{X} -groups. Many results have been obtained on minimal non- \mathfrak{X} -groups, for various choices of \mathfrak{X} . In particular, in [1] and [2] Belyaev and Sesekin characterized minimal non- FC -groups when they have a non-trivial finite or abelian factor group. They proved that such minimal non- FC -groups are finite cyclic extensions of divisible p -groups of finite rank, where p is a prime. Here we prove the following result.

Theorem 1. *Let G be a group that has a proper subgroup of finite index. Then G is a minimal non- M_rC -group if, and only if, G is a minimal non- FC -group.*

This is a joint work with Mounia Bouchelaghem from the University of Setif.

- [1] V. V. Belyaev: *Minimal non-FC-groups*, VI All Union Symposium Group Theory (Čerkassy, 1978), (Naukova Dumka 221, 1980) 97-102.
- [2] V. V. Belyaev, N. F. Sesekin: *Infinite groups of Miller-Moreno type*, Acta Math. Hungar. 26 (1975) 369-376.

Permutation modules of finite p -groups

T. Weigel

Let G be a finite p -group for some prime number p . In several disciplines in algebra it is important to understand whether a given finitely generated left $R[G]$ -module M , R a commutative ring with identity, is an $R[G]$ -permutation module, i.e., there exists a subgroup H of G such that M is isomorphic to $R[G/H]$. In this short talk we discuss this problem in the case when R is the ring of p -adic integers, and describe some applications in group theory.